Effect of Number of Contributed Modes on Accuracy of Multi-Modal Pushover Seismic Analysis of Irregular Frames

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Abstract
One of well-established static procedures for seismic evaluation and design of building structures in the equivalent nonlinear static procedures is Modal Pushover Analysis developed by Chopra and Goel (2002). For evaluation of effect of number of contributed modes on the accuracy of MPA procedure, 22 vertically irregular 5-story steel moment resisting frames are considered that each irregular frame is designed to represent low and high values of response reduction factor (R). Many nonlinear time histories and static pushover analyses are performed and the results of nonlinear static analyses are compared with the results for nonlinear dynamic analyses to evaluate the accuracy and conservatism of MPA for predicting target displacement, base shear, and story drifts.

Keywords: Equivalent nonlinear static analysis, Irregular frame, Modal pushover analysis, Seismic modeling, Steel moment resisting frame

1. INTRODUCTION

The structural engineering community has developed a new generation of design and a seismic evaluation procedure that incorporates performance based engineering concepts. It has been recognized that damage control must become more explicit design consideration. This aim can be achieved only by introducing some kind of nonlinear analysis into the seismic design methodology. In a short term, the most appropriate approach seems to be a combination of the nonlinear static analysis (NSP) and the response spectrum approach [1]. In the simplest case, the most important step process of seismic design, is the conception of an effective structural system that needs to be configured with due regard to all important seismic performance objectives, ranging from serviceability considerations to life safety and collapse prevention. This step comprises the art of seismic engineering, since no rigid rules can, or should, be imposed on the engineer’s creativity to devise a system that not only fulfills seismic performance objectives, but also pays tribute to functional and economic constraints imposed by the owner, the architect and other professionals involved in the design and construction of a building. [2] NSP is widely used in recent years for practical evaluation of seismic demands and for structural design. Recently, a new method of analysis, called Modal Pushover Analysis (MPA), was introduced by Chopra and Goel [3]. Based on the assumptions that the response of a structure is controlled by a single mode and the shape of that mode remains constant with time, MPA can lead to good estimates of the seismic demands of a building [4].

The seismic response of vertically irregular building frames, which has been the subject of numerous research papers, started getting attention in the late 1970s. A large number of papers have focused on plan irregularity resulting in torsion in structural systems. Vertical irregularities are characterized by vertical discontinuities in the distribution of mass, stiffness and strength. Very few research studies have been carried out to evaluate the effects of discontinuities in each one of these quantities independently, and majority of the studies have focused on the elastic response. In setback structures there is a sudden change in the vertical distribution of mass, stiffness, and in some cases, strength. A setback structure is thought of being made up of two parts: a base (the lower part having many bays), and a tower (the upper part with fewer bays) [5].

A major challenge to performance-based seismic design and engineering of buildings is to develop simple, yet effective, methods for designing, analyzing and checking the design of structures so that they reliably meet the selected performance objectives. Needed are analysis procedures that are capable of predicting the demands - forces and deformations - imposed by earthquakes on structures more realistically than has been done in building codes [3].

Previous investigations of MPA’s accuracy and efficiency dealt with regular frames [5]. Very recent works by Chintanapakdee and Chopra [7] address the effects on floor displacements, story drifts and plastic hinge rotations of ‘vertically’ irregular frames [4]. This study represents a further attempt to evaluate the
effect of contributed modes on accuracy of MPA for target displacement, interstory drift and base shear for the case of frames with setback irregularity, by comparing results to those obtained with nonlinear time history analysis (NL-THA).

2. DISCRIBTION OF MODELING AND GROUND MOTIONS

Models In this study, are the irregular moment resisting frame from Mazzolani [8] research that they was loaded by iranian building standard NO. 2800 [9] and designed as intermediate moment resisting frame (IMRF) using AISC-LRF99 code [10]. Each irregular frame is designed to represent low and high values of response reduction factor (R), therefore there are 44 models and here they are called in two classes, class I, the models from number 0 to 21 that are designed by low value of R and class II, the models from number 22 to 43 that are designed by high value of R. These frames (figure 1) are having three 10 meter spans and five stories. The height of first story is 3.4 meter and others have identical height of 3.6 meter. In these frames all beams and columns are assumed to be at their strong axis and connect to each other by welding. The inelastic analyses have been performed using static and dynamic structural analysis program IDARC2D [11] which is developed for nonlinear analysis of reinforced concrete and steel structures under static and dynamic loading. This program use fiber approach for inelastic analysis. Time history analyses employ fourteen (7 couples) natural records referred to as W1 to W14 in table 1. These accelerograms belong to strong ground motions that had been recorded in soil type C in USCGS category and having same specialty such as magnitude and distance.

In order to apply NL-THA for evaluation of nonlinear static procedure, scaling of ground motion records is required. The technique of scaling suggested by NEHRP (FEMA356) [12] is used here with elastic
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Spectrum defined in Iranian building standard [9] for soil type C. The scaled PGA of these records is shown in last column of table 1 and the scaled spectra are plotted in figure 2.

![Figure 2: Scaled acceleration spectrum of ground motions.](image)

### 3. NONLINEAR STATIC AND DYNAMIC RESULTS

Two types of nonlinear static and nonlinear dynamic analyses have been performed using the frame described earlier for 14 ground motions (7 pairs) and nonlinear static pushover analysis up to the MPA predicted target displacement, base shear and story drift for all models. Over 3000 nonlinear time histories and static pushover analyses are performed. The results of modal push over analysis with considering different modes are compared with the results for nonlinear dynamic analyses to evaluate the accuracy and conservatism of MPA.

#### 3.1. Errors and Correlation Definition

For evaluation of NSP purposed by MPA, one should determine the errors in resulted responses with regard to the same one which obtained from NL-THA in that ground motion. So determination of average, median and maximum error for this method is important. In this study for purpose of error calculation relative error is used as the accuracy indicator for this method of analysis. Relative error has illustrated in equation (1) in percent (1).

\[
\text{error} (\%) = \left( \frac{Q_{NL-THA} - Q_{MPA}}{Q_{NL-THA}} \right) \times 100
\]  

(1)

In these equations \(Q_{NL-THA}\) is the nonlinear time history response (such as target displacement, base shear, drift and etc.) for the \(i\) th ground motion and \(Q_{MPA}\) is the analogous response, resulted from nonlinear static procedure for \(i\) th ground motion. For evaluation of outcome results we need a parameter to verify the dependency or independency of the results. For this purpose statistics represents a non dimensional parameter which it is known as correlation factor. For a band including n-couple results, equation (2) may be used to calculate this factor.

\[
\rho = \frac{\sum_{i=1}^{n} (Q_{NL-THA} - \bar{Q}_{NL-THA})(Q_{MPA} - \bar{Q}_{MPA})}{\left[ \sum_{i=1}^{n} (Q_{NL-THA} - \bar{Q}_{NL-THA})^2 \right]^{1/2} \left[ \sum_{i=1}^{n} (Q_{MPA} - \bar{Q}_{MPA})^2 \right]^{1/2}}
\]  

(2)

In the recent equation \(Q_{NL-THA}\) and \(Q_{MPA}\) are define as in equations (1) and (2) and \(\bar{Q}_{NL-THA}\) is used as the average of nonlinear time history results and \(\bar{Q}_{MPA}\) defines the average of nonlinear static results. It is clear that if \(\rho = \pm 1\) the complete linear correlation is approved.

To illustrate the accuracy and the correlation rate of this approximate method for predicting of the responses, NL-THA responses resulted from nonlinear dynamic analyses are plotted versus MPA responses.
obtained from nonlinear static analyses to each of the 14 ground motions in all models which will be presented in relative following text.

3.2. Review of Displacement Results

Figure 3 presents the correlation ratio of target displacement for all analyses that including 616 nonlinear time history results versus 616 results obtained from more than 1800 nonlinear static analyses (determined by MPA considering at least three modes) is equal to 0.8094 Correlation value which is obtained shows the high accuracy of the MPA for prediction of peak inelastic displacement of buildings in a certain earthquake. By the way the graph shows the scatter rate of this response.

As mentioned before for investigation of the accuracy of this response, one can use relative error. Relative errors for this response are shown in figure 4 in percent. In this figure each bar shows the relative error interval for all ground motions and it has been existed for each model separately. So the maximum of relative error occurred in all ground motion for a certain model, is appeared at the top or bottom of the bar and the median value of relative error is shown by the dash. In figure 4, observe that error domain intermediate maximum error and minimum error in class I models, are between -30 to +40 percent whenever, in class II models are between -60 to +50 percent. Tolerance of median errors in all models is between -15 to +15 percent.

3.3. Review of Base Shear Results

Figure 5 presents the correlation ratio of base shear for all analyses that including 616 nonlinear time history results versus 616 results obtained from more than 1800 nonlinear static analyses (determined by MPA considering at least three modes) is equal to 0.9660 Correlation value which is obtained shows the high accuracy of the MPA for prediction of peak inelastic displacement of buildings in a certain earthquake. By the way the graph shows the scatter rate of this response.

For investigation of the accuracy of base shear response, the relative errors occur in estimation of this response are shown in figure 6 in percent. In this figure each bar shows the relative error interval for all ground motions and it has been existed for each model separately. So the maximum of relative error occurred in all ground motion for a certain model, is appeared at the top or bottom of the bar is shown by...
the dash In figure 6, observe that error domain intermediate maximum error and minimum error are variant and by the way are between -25 to +45 percent.

3.4. Review of Inter Story Drift Results

One of the other seismic responses which can be affected by irregularity in buildings is inter story drift. It should be noted that by moving from global responses such as target displacement toward local response like inter story drift leads to less accuracy in prediction of responses and this limitation is in the nature of pushover analysis as be shown in previous research [13]. To finding the ability of this method (NSP) for prediction of inter story drift, the values of NSP for this response is plotted versus resulted of NL-THA in figure 7. This figure is consisted of 3080 Cartesian points and it has contain the NSP and NL-THA drift for each story of 44 frame models in all of 14 ground motion. The correlation factor of this response is 0.8150 and as expected it shows good accuracy for prediction of this response.

For studying of the accuracy of story drift response, the relative errors occur in estimation of this response are shown in figure 8 in percent. Figure 8 presents the error domain intermediate maximum errors and minimum errors are different for class I and class II models as in class I models are between 0 to +45 percent while in class II models are between -60 to +50 percent. Also median errors are between 0 to +40 percent.

4. Effect of Number of Contributed Modes on correlation factor

Due to evaluation of effect of number of contributed modes on correlation factor, this factor computed for different responses and two class models separately by MPA procedure with including only one mode that called first mode method and MPA procedure with including necessary modes to reduce errors, compared in figure 9. In this figure shown that most effect of increasing contributed modes on correlation factor existed in drift responses and specially for models class I (low R).

![Figure 7: Scatter plot of estimated frame drifts.](image1)

![Figure 8: Relative errors of estimated frame drifts.](image2)

![Figure 9: Comparing correlation factor in first mode and MPA procedure.](image3)
4.1. Effect of Number of Contributed Modes on Accuracy of MPA results

The MPA determined total responses by combining the peak “modal” responses using the SRSS Combination rule of Eq. (3). Observe that errors tend to decrease as response contributions of more “modes” are included. Indeed, including more modes in MPA procedure increase accuracy, however determination of responses in each mode needs to more calculation but in this study specified which there are effectual number of modes for each response that MPA results has an acceptable accuracy.

\[ r_p \approx \left( \sum_{n=1}^{N} r_{n}^2 \right)^{1/2} \]  \hspace{1cm} (3)

, Where “r” is modal static response.

4.2. Effect of Number of Contributed Modes on Target Displacement

By plotting target displacement maximum and median relative error reduction percent from 1 mode including to 2 modes and 2 modes to 3 modes including illustrated in figure 10 and figure 11 respectively. Error reduction percent is different in target displacement maximum and median error. For target displacement maximum error and in the class I models, with increasing 1 to 2 modes including, approximately between 1 to 8 percent, error reduction occurred while in the class II models, is between 1 to 3 percent. Also for target displacement median error reduction is similar in all models and is between 1 to 7 percent. Remarkable point that shown in these figures is that error reduction for 2 to 3 modes in maximum and median error is very low, therefore, concluded for calculation of target displacement at least 1 modes including are sufficient.

4.3. Effect of Number of Contributed Modes on Base Shear

By plotting base shear maximum and median relative error shear reduction percent from 1 mode including to 2 modes and 2 modes to 3 modes including illustrated in figure 12 and figure 13 respectively. Error reduction percent is similar in base shear maximum and median error. In the class I models, with increasing 1 to 2 modes including, approximately between 10 to 20 percent, error reduction occurred while in the class II models, is between 10 to 40 percent. Another point that shown in these figures is that error reduction for 2 to 3 modes in maximum and median error is significant and are between 1 to 10 percent, therefore, in calculation of base shear by MPA, at least 3 modes should be considered for deriving better results.
4.3. Effect of Number of Contributed Modes on drift

By plotting drift maximum and median relative error reduction percent from 1 mode including to 2 modes including illustrated in figure 14 and figure 15 respectively. Error reduction percent is different in drift maximum and median error. For drift maximum error, in the class I models, with increasing 1 to 2 modes including, approximately between 1 to 10 percent, error reduction occurred while in the class II models, is between 5 to 20 percent. For drift median error, in the class I models, with increasing 1 to 2 modes including, approximately between 1 to 10 percent occurred while in the class II models, is between 5 to 35 percent. Important point that shown in these figures is that error reduction for 2 to 3 modes in maximum and median error is significant for median error in class II models and are between 1 to 10 percent, therefore, in estimation of drift by MPA, at least 2 modes should be considered for extract more accurate results.

![Figure 14: Effect of contributed modes on drift maximum error.](image)

![Figure 15: Effect of contributed modes on drift median error.](image)

5. Conclusions

It is used about 3000 nonlinear dynamic and static analyses in this study for evaluation of the accuracy and conservatism of the nonlinear static procedure known as modal push over for predicting the seismic responses of 22 irregular frames (having set-back) under 14 (7 coupled) scaled ground motion. Results of MPA analyses using only first mode can be summarized as follows:

- The correlation value is 0.81, 0.96, and 0.81 for frame roof displacement, base shear, and story drifts, respectively.
- The maximum median error is 18%, 55% and 50% for estimated frame roof displacement, base shear, and story drifts, respectively.
- For almost all frame models and all earthquakes, the MPA estimated base shears are in conservative. Estimation error for base shear increases with increase of R value.

From the results of multi-modal MPA analyses following conclusions can be made:

- The most effect of increasing contributed modes on correlation factor existed in drift responses and especially for frame models with low R.
- Efficient number of modes for estimating frame roof displacement, base shear, and story drifts, are 1, 3 and 2, respectively.
- Contribution of including larger number of modes more efficiently increases accuracy of MPA procedure for frames with higher R values.

It seems that this method can accurately predict the target displacements although the results are a little in conservative and using the MPA for the purpose of designing provides good estimation of target displacement but inadequate prediction of other demands such as a shear and inter story drift for such buildings having setbacks. The errors appear in this method for irregular building might be due to forming local mechanism in frame. According to inherent approximation of NSP, it can be reduced in shear and drift responses by considering more than two modes. Also the effect of variation of models dynamic properties, story stiffness and story strength, on accuracy of MPA results and relations of errors with these properties are important and should be studied in future works.

6. References


