Flexibility based method for the extent of damage in degrading bridge structures after in-service loading

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Abstract
This paper presents a method to identify damage in bridge structures based on the flexibility matrices in the modal strain space. In the study by the Ritz vectors extracted from flexibility matrix, the damage is identified. The localization approach is applied to low levels of damage on the prestressed concrete girder of simply supported bridge, with a focus on using a small number of sensors and only the fundamental mode of vibration. The validity of the method is also demonstrated using experimental modal data of a plate girder from I-40 Bridge over the Rio Grande. The predictions were found to closely match the actual response of the bridges. The proposed method can detect the damage in bridge structures using a limited number of sensors and vibration modes.

Keywords: Bridge, Damage, Flexibility, Modal strain, Ritz Vectors.

1. INTRODUCTION

Damage detection and health monitoring of large-scale structures are important challenges to engineering research. One common approach is to employ vibration characteristics of a structure to predict the damage locations and to estimate the amount of damage. However, it has been shown that damage in an early stage may not cause significant changes in the modal parameters. To overcome the sensitivity problems of modal parameters to damage, several alternative methods have been proposed. Among these methods, it is well recognized the methods that utilize the mode shapes are more efficient in damage identification. To show the damage localization and severity, the mode shapes must be analyzed and rearranged in appropriate form [1,2]. For this purpose, the methods such as strain mode shapes, strain energy and flexibility matrices have been proposed. Pandey et al. [3], showed that damage occurred in the structure, led to local changes in the shape of curvature mode shapes. Salawu and Williams [4] evaluated the performance of some procedures for locating damage using mode shape curvatures. Mode shape curvatures that introduced as modal strains are widely used to damage identification methods. Wahab and DeRoeck [5], investigated the curvature damage factor in a prestressed concrete bridge. Maeck and DeRoeck [6], calculated the bending and torsional stiffness changes using modal curvatures or modal strains. The strain based approaches require the direct measurement of dynamic strains or computing the second derivatives of measured modal displacements. In these methods, the noise induced by the measurement causes the errors in numerical procedures to compute the curvature of mode shapes. In addition, some of numerical methods for obtaining modal curvatures lead to inaccurate results. Maeck [7], proposed an optimization method to calculate the curvatures and removing the noises from experimental data. By using Maeck method, the modal strains are obtained with good accuracy and the boundary conditions can be imposed.

Dynamically measured flexibility matrix indices have also been proposed for damage detection of structures. The flexibility based identification methods have advantages in structural health monitoring. The flexibility matrix can calculated easily from the measured first few modes of the structure, without needing the numerical procedures. The formulation of the flexibility matrix is approximate. Because of the inverse relationship to the square of the modal frequencies, this approximation yields promising results [8]. The damage index can compute by comparing the flexibility matrices in the pre- and post-damaged states. Some of the methods used for this purpose are heuristic, conceived for particular types of the structures, and few can operate with multiple damage scenarios and with an arbitrary number of sensors [9]. Pandey and Biswas [10] presented damage detection and localization method based on the flexibility changes in the structure.
They computed the maximum absolute value of flexibility changes in the pre- and post-damaged states. Sohn and Law [11] used the extracted Ritz vectors for damage detection of a grid-type bridge model. The Ritz vectors are extracted from a flexibility matrix constructed using measured vibration data. According to Nour-Omid and Clough [12], the response quantities can be approximated more effectively by a smaller number of the Ritz vectors than the modal vectors.

In this paper the flexibility damage index based on the flexibility matrix and the Ritz vectors is employed in the modal strain space. The proposed method is verified through the experimental data from a prestressed concrete girder of simply supported bridge and the north plate girder from I-40 Bridge over the Rio Grande. The modal test data were processed and the damaged areas were identified by the proposed method. The damaged areas correspond well with the results obtained by the proposed damage detection method.

2. THEORETICAL BACKGROUND

Since the flexibility matrix can easily and accurately be estimated from the first few modes of vibration of the structure, many efforts have been made to find the damage indices using this matrix. The components of the flexibility matrices are deformations corresponding to static forces of unit magnitude acting at the coordinates of a structure. The flexibility matrix is divided into two parts: the modal flexibility matrix $F_m$ which is formed from the measured frequencies and modal vectors, and the residual flexibility $F_r$ formed from unmeasured residual modes [8].

$$ F = F_m + F_r = \phi^T m \phi + \phi^T \Lambda \phi $$

where, $\Lambda$ is the spectral matrix. The main diagonal of $\Lambda$ contains the square of the modal frequencies. The $\phi$ vectors are mass-ortho-normalized modal vectors. Because of the inverse relationship to the square of the modal frequencies, the complete flexibility is approximated by the modal flexibility matrix and the contribution of the residual flexibility which is generally about 3-10% of the modal flexibility matrix is ignored [8].

The Ritz vectors are extracted from the flexibility matrices. They have many potential advantages in structural dynamics over modal parameters. However, very few studies have applied Ritz vectors to damage detection problems. These vectors provide a better understanding basis for damage detection problems. Using the modal flexibility matrix $F_m$, the first Ritz vectors can be computed as:

$$ F_r^1 = F_m f $$

where $f$ is a spatial load distribution vector. Since the modal inertia forces are proportional to the mode shapes, the mode shapes are substituted for spatial load distribution vector in Eq.2. Due to the modal strain advantages, in this paper all of the modal displacements (mode shapes) are converted to the modal strains. Modal strains can be obtained by mixed approach proposed by Maeck [7] or Spline method. The mixed approach is an optimization method for calculating the curvatures and removing the noises from experimental data. By using mixed approach, the modal strains are obtained with good accuracy and the boundary conditions can be imposed [7]. Spline method is a mathematical approach that frequently used in CAD (Computer Aided Design) applications [7]. The Spline bending functions generate a single piecewise parametric polynomial function through any number of control points of the mode shapes and the modal curvatures can be calculated from the Spline by analytical approaches.

In the study, the modal strain vectors are substitute for the mode shape vectors in Eqs. (1) and (2). The absolute difference of the first Ritz vectors in the modal strain space, from pre- and post-damage states forms the damage patterns of the beam structure. The candidate damage elements are identified from the damage pattern. The damage patterns formed by Ritz vectors in the modal strain space do not yield accurate results near the boundary regions and the method does not identify the exact location of the damage; only the candidate damaged elements are determined. Thus, the method is improved here by proposing a new algorithm based on the statistical approaches. In the statistical approaches the damaged elements are assumed as random variables. To determine the exact location of damage, the unwanted variables must be removed from the domain. For beam type structures, three sub domains exist through degrees of freedom; one middle sub domain and two end sub domains. In free vibration tests the two end sub-domains are at a distance of 0.224L from the ends (theoretical nodal points of the first bending vibration mode).

The random variables of the two end sub domains are excluded in the statistical analyses and only the middle part of the beam is considered. Using the Hypothesis test, a damage index for each degree of freedom of the beam is obtained. Here, the null Hypothesis ($H_0$) and alternate hypothesis ($H_1$) are defined. $H_0$ corresponds to element $j$ of the structure that is not damaged and $H_1$ corresponds to element $j$ of the structure that is damaged. Before employing the Hypothesis test, the damage indices are standardized using Eq.3 [13]:

$$ Z_j = \frac{Y_j - \mu_j}{\sigma_j} $$

where $Y_j$ is the damage index, $\mu_j$ and $\sigma_j$ are the mean and standard deviation of the $j$th damage index, respectively. The Hypothesis test is performed for each degree of freedom and the damage index is calculated as:

$$ H_0: Y_j < \mu_j $$

If the Hypothesis test is rejected, the damaged area is identified. The damaged area is determined as the region where the Hypothesis test is rejected and the damage index is greater than $\mu_j$.
$$\beta_j = j \cdot z \beta_j - \mu$$

where $z_j$ is the standardized damage index for $j$th element, $\mu$ is the mean of $\beta_j$'s and $\sigma_\beta$ is the standard deviation of $\beta_j$'s. In this study, the absolute changes of the first Ritz vectors from pre- and post-damaged states are substituted for $\beta_j$'s. The decisions are made based on $z_{\eta}$, a threshold level of damage and can be considered as a discriminating level [13]. The symbol $\eta$ represents level of significance of the test. By comparing the normalized damage index $z_j$ to $z_{\eta}$, the damaged elements are determined. In the study the value of $z_{\eta}$ is 1. If $z_j < z_{\eta}$ null Hypothesis (H0) is chosen. When $z_j > z_{\eta}$, the element is damaged and the alternate Hypothesis is chosen. The proposed method is defined as an algorithm in Fig. 1. The proposed algorithm is verified through experimental damage simulation of prestressed concrete girder and the plate girder of I-40 Bridge.

3. EXPERIMENTAL STUDY ON PRESTRESSED CONCRETE GIRDER

Prestressed concrete girders are frequently used in bridge construction; however, a reliable and efficient assessment method for the deterioration and damage of prestressed concrete girders is not yet available. The experiments on the full-scale prestressed concrete bridge girders to simulate real conditions are necessary. For the purpose, Saskatchewan Highways and Transportation Administration donated a number of prestressed concrete bridge girders that were reclaimed form a dismantled bridge, to the laboratories. Several of these girders were instrumented and tested in the Structures Laboratory at the University of Saskatchewan [14]. In the study, the experimental modal data of one of these prestressed concrete bridge girders are analyzed by the proposed method. The specimen included a simply supported, full-scale prestressed concrete girder removed from an abandoned bridge. The girder was 12.2 meters long, spanned 11.9 meters, and had a 1216 x 508 mm cross section, as shown in Fig. 2 and Fig. 3-b. Experiments were carried under well controlled conditions in the Structural Laboratory in the College of Engineering at the University of Saskatchewan. In the experiments, the girder was simply supported at its four corners. Damage was simulated by removing small square blocks of concrete (150 x 150 mm in plan and 35 mm deep) from the top surface of the specimen as shown in Fig. 2. It should be noted that this was a very low level of damage for the bridge girder, corresponding to a local reduction in flexural rigidity of approximately 2.49 %. The Young’s modulus (E=26.1 GPa) was calculated based on the compressive strength of 34.5 MPa for the concrete, a value which was indicated on the design drawings for the girder.
The experimental setup for the prestressed concrete girder is shown in Fig.3-a. The hydraulic shaker was mounted on the centre of the girder, while both the accelerometers and strain gauges sets were positioned at six evenly spaced locations along each side of the girder. The girder was simply supported at both ends. One end was supported on a steel angle, allowing only freedom of rotation for the girder and preventing translations. The other end was supported using a roller, allowing the girder freedom of both rotation and longitudinal movement. The experimental procedure consisted of measuring the initial (undamaged) dynamic properties of the system, and then inducing a new state of damage and measuring the properties associated with the damage state [14]. In the study, the results indicate that the use of a relatively small number of measurement points to characterize the fundamental mode shape is sufficient to detect and localize damage with a reasonable level of accuracy. Then, only the fundamental vibration characteristics of the prestressed concrete girder are analyzed. The fundamental frequency and mode shape of the girder changed slightly after the damage (Table 1 and Fig. 4-a).

Table 1-Fundamental frequency of the prestressed concrete girder

<table>
<thead>
<tr>
<th></th>
<th>Fundamental frequency before damage</th>
<th>7.56134 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fundamental frequency after damage</td>
<td>7.56008 Hz</td>
</tr>
</tbody>
</table>

![Figure 3. Experimental setup and cross section of the prestressed concrete bridge girder [14]](image)

The fundamental vibration characteristics of the prestressed concrete girder analyzed by the proposed method summarized in Fig. 1. Due to the coarse mesh of the measurement points, before analyzing the mode shape, the measurement points were increased to 81 points by Spline interpolation functions. The modal strains obtained by mixed approach [7]. The first Ritz vector extracted from the flexibility matrix and special load distribution vector in the modal stain space. The damage pattern obtained by the first Ritz vector and the inaccurate variables near the supports were removed from the damage pattern. By using the Hypothesis test described in the section 2 of the paper, the damage index of the girder was determined (Fig. 4-b). According to Fig. 4-b, the predicted damaged areas correspond well with two damaged area of the girder. The proposed method can localize the damaged areas of the concrete girders in the early sage of damage.

![Figure 4. Fundamental mode shape and damage localization of the prestressed concrete bridge girder](image)
4. FEILD STUDY ON NORTH PLATE GIRDER OF I-40 BRIDGE

To evaluate the feasibility of the proposed method, the field data from the simulated damage on I-40 Bridge over the Rio Grande are used here. The elevation view of the portion of the I-40 Bridge for the modal testing, analysis and damage identification studies is shown in Fig.5. The north bound and south bound of I-40 Bridges over the Rio Grande in Albuquerque, New Mexico, were demolished in 1993 and were replaced by a new bridge [15]. The spans of each bridge consisted of a concrete deck supported by two welded, steel, plate-girders and three steel stringers. The stringers transferred loads from the deck to the plate girders via floor beams located at 6.096 m (20 ft) intervals. Cross-bracing was provided between the floor beams. During the summer of 1993, New Mexico State University and Los Alamos National Laboratory introduced sequential damage to the bridge to simulate fatigue crack growth in the main plate girder of one of the bridges in order to test various damage identification methods. To simulate fatigue cracking in the bridge, four levels of damage were introduced to the middle span of the north plate girder [15]. Damage was introduced by making various torch cuts in the web and flange of the girder (Table 2). In this paper, data collected from the mentioned first through fourth levels of damage is used to evaluate the field applicability of the proposed method.

A set of forced vibration tests was performed on the structure before any simulated damage. For each sequential damage scenario, forced vibration tests were repeated. The excitation source was a hydraulic shaker consisting of a 96.526 kN reaction mass supported by three air springs on top of drums filled with sand. The input force was provided by a 9.7861 kN hydraulic actuator bolted under the center of the mass. A random-signal generator was used to produce a 8.8964 kN peak-force uniform random signal over the frequency range 2–12 Hz. An accelerometer mounted on the reaction mass was used to measure the force input-time history. The output acceleration time histories in the vertical direction were measured by 11 Endevco 7751-500 accelerometers. The accelerometers were mounted on the inside web of the plate girder at mid-height of the plate girder with a nominal spacing of 4.8768 m (16 ft) as shown in Fig. 6. Mode shapes were determined from the cross-spectra of the various accelerometer readings [15]. Since modal data for the undamaged condition of the bridge are available, no attempt is made to build a numerical model of a baseline structure.

![Figure 5. Elevation view of the portion of I-40 bridge that was tested [15]](image)

<table>
<thead>
<tr>
<th>No. of Scenario</th>
<th>Damage description</th>
<th>Frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Undamaged</td>
<td>Mode 1 2.48 Mode 2 2.96 Mode 3 3.50 Mode 4 3.08 Mode 5 4.17 Mode 6 4.63</td>
</tr>
<tr>
<td>1</td>
<td>61 cm cut at the center of the web</td>
<td>Mode 2 2.52 Mode 3 3.0 Mode 4 3.57 Mode 5 4.12 Mode 6 4.21</td>
</tr>
<tr>
<td>2</td>
<td>182 cm cut in the web to the bottom flange</td>
<td>Mode 2 2.52 Mode 3 2.99 Mode 4 3.52 Mode 5 4.09 Mode 6 4.19</td>
</tr>
<tr>
<td>3</td>
<td>182 cm cut of the web and half of bottom flange cut</td>
<td>Mode 2 2.46 Mode 3 2.95 Mode 4 3.48 Mode 5 4.04 Mode 6 4.14</td>
</tr>
<tr>
<td>4</td>
<td>182 cm cut of the web and entire bottom flange cut</td>
<td>Mode 2 2.30 Mode 3 2.84 Mode 4 3.49 Mode 5 3.99 Mode 6 4.15</td>
</tr>
</tbody>
</table>
Damage is associated with decrease in the eigenfrequencies, increases in the damping values and alternations of the modes of vibration of the structure. According to Table 2, no significant changes occurred in the eigenfrequencies of the north plate girder. The eigenfrequencies have shown little promise for detecting the presence of damage. As observed in Fig. 6, the set of accelerometers is very coarse. Due to the coarse set of measurement points, the modal stains can not obtained with a reasonable accuracy. To overcome this problem, by using the cubic Spline interpolation functions, the measurement points are increased. By increasing the measurement points and using the Spline method, the modal strains are obtained with a reasonable accuracy.

In the study to show the feasibility of the proposed method, the fundamental vibration characteristics of the north plate girder of I-40 Bridge are utilized. The measurement points of span 1 and span 3 were increased to 27 and the measurement points of span 2 were increased to 41 points by Spline interpolation functions. The modal strains of the each span of the girder were obtained by Spline method. The damage patterns of each span through four damage scenarios are identified by the flexibility matrix and the Ritz vectors in the modal strain space (Fig. 7).

According to Fig. 7, the damage patterns of span 1 and 3 can not detect damaged area. The damage patterns of span 2 are convex in Fig. 7-a and Fig. 7-b. The peaks of the convexities intend to the damage location of the north girder. By using the statistical approach described in section 2, the damage will be localized in the span 2 of the girder. The damage severity is also determined by the magnitude of the Ritz
vectors in the modal strain space (step 3 of the proposed algorithm in Fig. 1). The damage severity of the north plate girder is shown in Fig. 8. Damage severity in the forth scenario is very high compares to first, second and third scenarios (Fig. 8-a). To show the damage severity of early stages, the diagrams associated with first three scenarios are presented in Fig. 8-b. According to Fig. 8, the proposed method can determine the location and the severity of damage from the early stage to the failure.

![Damage index Measurement](image)

**Figure 8. Damage localization and severity estimation through four damage scenarios on the north plate girder of I-40 Bridge**

5. **CONCLUSIONS**

In this paper, a new damage detection algorithm has been proposed based on the flexibility matrices in the modal strain space. The proposed method utilizes Ritz vectors generated by flexibility matrix in the modal strain space. By applying Hypothesis test on the damage patterns obtained by the Ritz vectors from pre and post damage states, the damage is well identified. The experimental verification studies presented in this paper show the validity of proposed method and the following conclusions are drawn:

- The proposed method can detect low levels of damage on the prestressed concrete girder with a focus on using a small number of sensors and only the fundamental mode of vibration.

- The validity of the method is also demonstrated using experimental modal data of a plate girder from I-40 Bridge over the Rio Grande. The predictions were found to closely match the actual response of a plate girder of I-40 Bridge through four damage scenarios.

- The proposed method can detect the damage in bridge structures using a limited number of sensors and vibration modes from early stage to the failure.

6. **REFERENCES**


