Sensitive Analysis on Effective Parameters in Breakwater Design against Wave Diffraction

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ABSTRACT

Beaches erode and the obvious environmental and landscape degradation of many coastal stretches are problems that coastal engineers are spending time solving. Different protection methods have been used over time, most on the basis of the artificial nourishment of beaches and the building structures such as groynes and detached breakwaters. Breakwaters are artificial structures, generally parallel to the coastline, inspired by the working of natural formations, protecting a certain stretch from wave action and being able to create accretion areas. This is why these structures have been in general use, with different results, since the 1970s in countries such as Japan, the United States, Spain, Italy, and Australia. Lakes coast confronts a wide range of natural hazards from severe storms, floods, landslides and shoreline erosion. All of these coastal hazards threaten both lives and property—a problem that becomes more pressing as the coastal population continues to rise. Coastal erosion, deposition, and flooding can also be exacerbated by lake level regulation, water diversion and coastal resource use. Structures like breakwaters always confront sea phenomenon like diffraction and refraction of waves. Diffraction, which be made by contact of water waves to breakwater structure and results almost concentric circles of waves, makes evaluating this phenomenon far more important on structures. By using the powerful software MIKE 21 in sea hydraulics problems, finding the optimized model of the breakwater arms can be obtained. Meanwhile other factors are playing role in this optimized model. By sensitive analysis of the main parameters such as entrance wave spectrum, wave period, porosity coefficient of breakwater and effect of spongy layers on the results, the partnership and affection of all parameters on the result of the model will be revealed. Presenting the optimized breakwater models against diffraction and Showing the most important parameters on the basis of sensitive analysis is the results of this research.

1. INTRODUCTION

Diffraction refers to various phenomena which occur when a wave encounters an obstacle. It is described as the apparent bending of waves around small obstacles and the spreading out of waves past small openings. While diffraction occurs whenever propagating waves encounter such changes, its effects are generally most pronounced for waves where the wavelength is on the order of the size of the diffracting objects. If the obstructing object provides multiple, closely-spaced openings, a complex pattern of varying intensity can result. This is due to the superposition, or interference, of different parts of a wave that traveled to the observer by different paths.

Diffraction arises because of the way in which waves propagate; this is described by the Huygens–Fresnel principle. The propagation of a wave can be visualized by considering every point on a wave front as a point source for a secondary radial wave. The subsequent propagation and addition of all these radial waves form the new wave front. When waves are added together, their sum is determined by the relative phases as well as the amplitudes of the individual waves, an effect which is often known as wave interference. The summed amplitude of the waves can have any value between zero and the sum of the individual amplitudes. Hence, diffraction patterns usually have a series of maxima and minima.

The form of a diffraction pattern can be determined from the sum of the phases and amplitudes of the Huygens wavelets at each point in space. There are various analytical models which can be used to do this.
including the Fraunhofer diffraction equation for the far field and the Fresnel Diffraction equation for the near field. Most configurations cannot be solved analytically, but can yield numerical solutions through finite element and boundary element methods.

In fluid dynamics, the Boussinesq approximation for water waves is an approximation valid for weakly non-linear and fairly long waves. The approximation is named after Joseph Boussinesq, who first derived them in response to the observation by John Scott Russell of the wave of translation (also known as solitary wave or soliton). The 1872 paper of Boussinesq introduces the equations now known as the Boussinesq equations.

The Boussinesq approximation for water waves takes into account the vertical structure of the horizontal and vertical flow velocity. This results in non-linear partial differential equations, called Boussinesq-type equations, which incorporate frequency dispersion (as opposite to the shallow water equations, which are not frequency-dispersive). In coastal engineering, Boussinesq-type equations are frequently used in computer models for the simulation of water waves in shallow seas and harbours.

The essential idea in the Boussinesq approximation is the elimination of the vertical coordinate from the flow equations, while retaining some of the influences of the vertical structure of the flow under water waves. This is useful because the waves propagate in the horizontal plane and have a different (not wave-like) behaviour in the vertical direction. Often, as in Boussinesq’s case, the interest is primarily in the wave propagation.

This elimination of the vertical coordinate was first done by Joseph Boussinesq in 1871, to construct an approximate solution for the solitary wave (or wave of translation). Subsequently, in 1872, Boussinesq derived the equations known nowadays as the Boussinesq equations.

2. EQUATIONS

The steps in the Boussinesq approximation are:

- a Taylor expansion is made of the horizontal and vertical flow velocity (or velocity potential) around a certain elevation,
- this Taylor expansion is truncated to a finite number of terms,
- the conservation of mass (see continuity equation) for an incompressible flow and the zero-curl condition for an irrotational flow are used, to replace vertical partial derivatives of quantities in the Taylor expansion with horizontal partial derivatives.

Thereafter, the Boussinesq approximation is applied to the remaining flow equations, in order to eliminate the dependence on the vertical coordinate. As a result, the resulting partial differential equations are in terms of functions of the horizontal coordinates (and time).

As an example, consider potential flow over a horizontal bed in the \((x,z)\) plane, with \(x\) the horizontal and \(z\) the vertical coordinate. The bed is located at \(z = -h\), where \(h\) is the mean water depth. A Taylor expansion is made of the velocity potential \(\phi(x,z,t)\) around the bed level \(z = -h\):

\[
\varphi = \varphi_b + z \left[ \frac{\partial \varphi}{\partial z} \right]_{z=-h} + \frac{1}{2} z^2 \left[ \frac{\partial^2 \varphi}{\partial z^2} \right]_{z=-h} + \frac{1}{6} z^3 \left[ \frac{\partial^3 \varphi}{\partial z^3} \right]_{z=-h} + \frac{1}{24} z^4 \left[ \frac{\partial^4 \varphi}{\partial z^4} \right]_{z=-h} + \cdots,
\]

\[
\varphi = \varphi_b - \frac{1}{2} z^2 \frac{\partial^2 \varphi_b}{\partial x^2} + \frac{1}{24} z^4 \frac{\partial^4 \varphi_b}{\partial x^4} + \cdots \Bigg[ z \left[ \frac{\partial ^2 \varphi}{\partial z^2} \right]_{z=-h} - \frac{1}{6} z^3 \frac{\partial^3 \varphi}{\partial z^3} \bigg|_{z=-h} + \cdots \Bigg] - \left( \varphi_b - \frac{1}{2} z^2 \frac{\partial^2 \varphi_b}{\partial x^2} + \frac{1}{24} z^4 \frac{\partial^4 \varphi_b}{\partial x^4} + \cdots \right),
\]

where \(\varphi_b(x,t)\) is the velocity potential at the bed. Invoking Laplace's equation for \(\varphi\), as valid for incompressible flow, gives:

\[
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0.
\]

This series may subsequently be truncated to a finite number of terms.

For water waves on an incompressible fluid and irrotational flow in the \((x,z)\) plane, the boundary conditions at the free surface elevation \(z = \eta(x,t)\) are:

\[3\]
\[
\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} - w = 0
\]
\[
\frac{\partial \varphi}{\partial t} + \frac{1}{2} \left( u^2 + w^2 \right) + g \eta = 0, \quad (2)
\]

where:
- \( u \) is the horizontal flow velocity component: \( u = \frac{\partial \varphi}{\partial x} \),
- \( w \) is the vertical flow velocity component: \( w = \frac{\partial \varphi}{\partial z} \),
- \( g \) is the acceleration by gravity.

Now the Boussinesq approximation for the velocity potential \( \varphi \), as given above, is applied in these boundary conditions. Further, in the resulting equations only the linear and quadratic terms with respect to \( \eta \) and \( u_b \) are retained (with \( u_b = \frac{\partial \varphi_b}{\partial x} \) the horizontal velocity at the bed \( z = -h \)). The cubic and higher order terms are assumed to be negligible. Then, the following partial differential equations are obtained:

**set A — Boussinesq (1872),**
\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \left[ (h + \eta) u_b \right] = \frac{1}{6} h^3 \frac{\partial^3 u_b}{\partial x^3}, \\
\frac{\partial u_b}{\partial t} + u_b \frac{\partial u_b}{\partial x} + g \frac{\partial \eta}{\partial x} = \frac{1}{2} h^2 \frac{\partial^2 u_b}{\partial t \partial^2 x}. \quad (3)
\]

This set of equations has been derived for a flat horizontal bed, i.e. the mean depth \( h \) is a constant independent of position \( x \). When the right-hand sides of the above equations are set to zero, they reduce to the shallow water equations.

Under some additional approximations, but at the same order of accuracy, the above set A can be reduced to a single partial differential equation for the free surface elevation \( \eta \):

**set B — Boussinesq (1872),**
\[
\frac{\partial^2 \eta}{\partial t^2} - gh \frac{\partial^2 \eta}{\partial x^2} - gh \frac{\partial^2}{\partial x^2} \left( \frac{3}{2} \frac{\eta^2}{h} + \frac{1}{3} h^2 \frac{\partial^2 \eta}{\partial x^2} \right) = 0, \quad (4)
\]

From the terms between brackets, the importance of nonlinearity of the equation can be expressed in terms of the Ursell number. In dimensionless quantities, using the water depth \( h \) and gravitational acceleration \( g \) for non-dimensionalization, this equation reads, after normalization:

\[
\frac{\partial^2 \psi}{\partial \tau^2} - \frac{\partial^2 \psi}{\partial \xi^2} - \frac{\partial^2}{\partial \xi^2} \left( \frac{1}{2} \psi^2 + \frac{\partial^2 \psi}{\partial \xi^2} \right) = 0, \quad (5)
\]

with:
- \( \psi = \frac{3 \eta}{h} \) : the dimensionless surface elevation,
- \( \tau = \sqrt{\frac{3}{h}} t \sqrt{\frac{g}{h}} \) : the dimensionless time, and
- \( \xi = \sqrt{\frac{3}{h}} x \) : the dimensionless horizontal position.

**Linear frequency dispersion**

Water waves of different wave lengths travel with different phase speeds, a phenomenon known as frequency dispersion. For the case of infinitesimal wave amplitude, the terminology is linear frequency dispersion. The frequency dispersion characteristics of a Boussinesq-type of equation can be used to determine the range of wave lengths, for which it is a valid approximation.

The linear frequency dispersion characteristics for the above set A of equations are:\[4\]
\[
c^2 = gh \frac{1 + \frac{1}{6} k^2 h^2}{1 + \frac{1}{2} k^2 h^2}, \quad (6)
\]
with:
c the phase speed,
k the wave number (k = 2π / λ, with λ the wave length).
The relative error in the phase speed c for set A, as compared with linear theory for water waves, is less than 4% for a relative wave number kh < ½ π. So, in engineering applications, set A is valid for wavelengths λ larger than 4 times the water depth h. The linear frequency dispersion characteristics of equation B are:[4]

\[ c^{3} = gh \left( 1 - \frac{1}{3} k^2 h^2 \right). \quad (7) \]

3. NUMERICAL MODELLING

Here there is a case study on a Port in south of Iran and East Bandar-Abbas which has its own aspects of design and optimization.

Firstly input files will be revealed then step by step designing procedure will be showed.

Input files:

![Figure 1 - Bathymetry Model](image1)

![Figure 2 - Bathymetry Model with 50 m lengthening the main arm of the breakwater.](image2)

![Figure 3 - Damping Model](image3)
Main results of this case study are as below:
Diffraction modeling result for 270 degree waves:

Figure 4 – Proximity Model

Figure 4 – Diffraction modeling result for 270 degree waves with 50 m lengthening of
the main arm:

Figure 5 - Diffraction modeling result for 270 degree waves with 100 m lengthening of
the main arm:

Figure 6 - Diffraction modeling result for 270 degree waves with 150 m lengthening of
the main arm:

Figure 7 - Diffraction modeling result for 270 degree waves with 150 m lengthening of
the main arm:
Figure 8 - Diffraction modeling result for 300 degree waves:

Figure 9 - Diffraction modeling result for 300 degree waves with 50 m lengthening of the main arm:

Figure 10 - Diffraction modeling result for 300 degree waves with 100 m lengthening of the main arm:

Figure 11 - Diffraction modeling result for 300 degree waves with 150 m lengthening of the main arm
Figure 12 - Diffraction modeling result for 330 degree waves with 50 m lengthening of the main arm:

Figure 13 - Diffraction modeling result for 330 degree waves with 100 m lengthening of the main arm:

Figure 14 - Diffraction modeling result for 330 degree waves with 150 m lengthening of the main arm:

Table 1 - Results

<table>
<thead>
<tr>
<th>Wave Orientation</th>
<th>Diffraction Coefficients in different conditions (Percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Existing condition</td>
</tr>
<tr>
<td>270</td>
<td>47</td>
</tr>
<tr>
<td>300</td>
<td>75</td>
</tr>
<tr>
<td>330</td>
<td>87</td>
</tr>
</tbody>
</table>

4. CONCLUSION

Regarding the Numerical model based on Mike 21 software results, it is concluded that based on numerical studies 50 m lengthening is much more efficient but on the other hand we will have site study in construction
period that will cover likely numerical model gaps and the output of the project will be as economic as it should like every other engineering projects.

5. ACKNOWLEDGEMENT

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6. REFERENCES

1. Sorensen R.M. (2006), Basic Coastal Engineering Vol.1