Application of Wavelet method in de-noising option prices

Kazem Nouri\textsuperscript{1} Masoumeh Zangian\textsuperscript{2}

Department of Mathematics, Faculty of Mathematics, Statistics and Computer Sciences, Semnan University, P. O. Box 35195-363, Semnan, Iran

Abstract

In so much financial time series are known to carry noise, elimination of noise is necessary. Due to multi-scaling property, the wavelet method is very efficient in dealing with noisy data series. In specific, we propose to use the wavelet method to de-noise option prices before estimating the option-implied risk neutral density (RND) and forecasting future option prices. We use of two RNDs estimated from the perturbed prices and the filtered prices to forecast the out-of-sample options, respectively. Moreover, we compare them with the true Black-Scholes option prices. Results of this study show that, through the use of Monte Carlo simulations, the power of the wavelet method in the de-noising of option price data. It is clearly seen that, by de-noising the perturbed option prices using the wavelet method, most of the noise is removed and the wavelet de-noising method is robust to different levels of noise variance.

Keywords and phrases: De-noise, Option pricing, RND, Wavelet analysis, Monte Carlo simulation.

1. Introduction

We can distinguish three types of applications of the wavelet method in economics and finance. In brief, those are applications which, (1) relate to the analysis of multi-scale problems; (2) relate to the estimation of unknown parameters of a model; and (3) relate to the removal of noise from raw data series. The goal of this paper relates to the third objective. As example of the first type of application, Ramsey and Lampart (1998a, b)[6], they use wavelets to analyze the relationship between expenditure and income, and between money and income at six different time scales. As example of the second type of application, Haven (2009)[4] use wavelets to estimate the risk-neutral moment generating function of the stochastic process followed by the underlying asset of a European option. The wavelet method is shown to perform very well in estimating unknown parameters. Finally, the third type of application, Sun

\textsuperscript{1}Corresponding author: knouri@semnan.ac.ir, knouri@iust.ac.ir
\textsuperscript{2}ezangian@yahoo.com
and Meinl (2012)[7] develops a new wavelet-based filtering algorithm, which works particularly well when jumps are present in financial time series. In this paper, we investigate how the wavelet method can be used to de-noise option prices in order to better estimate risk-neutral densities and hence to provide more accurate forecasts for future option prices.

2. Methodology

2.1. Wavelet Decomposition

A key feature of the wavelet transform is that it can decompose any square integrable function \( y(t) \) into a combination of some scaling functions \( \phi_{M,n}(t) \) and wavelet functions \( \psi_{M,n}(t) \).

\[
y(t) = \sum_{n=-\infty}^{\infty} S_{M,n} \phi_{M,n}(t) + \sum_{m=-\infty}^{M} \sum_{n=-\infty}^{\infty} T_{m,n} \psi_{m,n}(t),
\]

Where \( \phi_{M,n}(t) = 2^{-M/2} \phi(2^{-M} t - n) \), \( \psi_{m,n}(t) = 2^{-m/2} \psi(2^{-m} t - n) \). \( M \) and \( m \) are scale parameters which control the dilation of the scaling and wavelet functions; \( n \) is the shift parameter which controls the translation of the scaling and wavelet functions. \( S_{M,n} \) are approximation coefficients. \( T_{m,n} \) are called detail coefficients. \( S_{M,n} = \int_{-\infty}^{\infty} y(t) \phi_{M,n}(t) dt \), \( T_{m,n} = \int_{-\infty}^{\infty} y(t) \psi_{m,n}(t) dt \).

As can be seen from the above integrals, the expansion formula is valid for a continuous signal \( y(t) \) of infinite length. In practice, signals are usually discrete and are of finite length. For a discrete signal \( y(t) \) of length \( N = 2^M \), which is an integer power of two, the discrete wavelet expansion is performed

\[
y(t) = \sum_{n=-\infty}^{2^{M-1}} S_{M,n} \phi_{M,n}(t) + \sum_{m=-\infty}^{M} \sum_{n=-\infty}^{2^{M-1}} T_{m,n} \psi_{m,n}(t).
\]

2.2. Wavelet De-noising

Wavelet de-noising is based on the wavelet decomposition. With hard thresholding, all wavelet coefficients less than a fixed positive constant in magnitude are set to zero and the remaining wavelet coefficients are kept unchanged:

\[
T_{m,n}^H = \begin{cases} 
0 & \text{if } |T_{m,n}| < \lambda \\
T_{m,n} & \text{if } |T_{m,n}| \geq \lambda
\end{cases}
\]

\[
T_{m,n}^S = \begin{cases} 
0 & \text{if } |T_{m,n}| < \lambda \\
\text{sign}(T_{m,n}) |T_{m,n} - \lambda| & \text{if } |T_{m,n}| \geq \lambda
\end{cases}
\]

They are carried out with the wavelet toolbox in Matlab. We use the Black-Scholes model (1973)[2] in this paper to generate option prices.

2.3. Experiment procedures

There are six steps in the experiment and they can be categorized into three parts:

Part 1:

1. For a given set of parameters \( \{F, K, T, r, \sigma\} \), simulate \( N \) Black-Scholes call options prices \( c = \{c_1, c_2, \ldots, c_N\} \) corresponding to \( N \) different strike prices. \( F \) is the underlying futures price; \( K = \{K_1, K_2, \ldots, K_N\} \) are \( N \) strike prices; \( T \) is the time-to-maturity; \( r \) is the risk-free interest rate, and \( \sigma \) is the volatility of the underlying asset.
2. Generate \( N \) normally distributed random numbers as noise.
In this paper, we use the Symmlet wavelet to de-noise implied volatilities in the perturbed option prices and substitute the de-noised volatilities back into the Black-Scholes formula to obtain the de-noised option prices $c_D$.

**Part 2:**

1. Estimate the RND from the perturbed option prices $c_P$ and the de-noised option prices $c_D$ respectively. The cubic spline method in Bliss and Panigirtzoglou (2002) [3] is used in this step to estimate the RND.

**Part 3:**

1. Simulate another set of Black-Scholes option prices $c$ from the parameters $\{F', K', T', r, \sigma\}$. The apostrophe indicates out-of-sample parameters.

2. Use the two RNDs estimated from the perturbed prices and the filtered prices to forecast the out-of-sample options, respectively.

### 2.4 Smoothed volatility smile-cubic spline method

For $N$ observed options, the market implied volatilities $\sigma_i$ and deltas $\Delta_i$, where $i = 1, 2, \ldots, N$. The objective function to be minimized is as follows:

$$
\min_\theta \{ \sum_{i=1}^N w_i (\sigma_i - \hat{\sigma}_i(\Delta|\theta))^2 + (1 - \alpha) \int_{\Delta_1}^{\Delta_N} \sigma''(\Delta|\theta)^2 d\Delta, \nonumber \}
$$

where $\theta$ denotes the vector of unknown cubic spline parameters; $w_i$ is the weight; and $\hat{\sigma}_i(\Delta|\theta)$ is the fitted volatility based on the estimated parameter $\theta$; $\sigma''(\Delta|\theta)^2$ is the squared curvature of the regression function $\sigma(\Delta|\theta)$. The solution to this minimization can be found in Lange (1998) [5].

### 3. The Monte Carlo simulation

Monte Carlo simulations are based on the FTSE-100 index options market. The option prices and strike prices are quoted in index points. We ignore deep in-the-money options since they are not frequently traded and their prices tend to be stale. Therefore, we will have 182 call options in total. $T$ is set to be 0.0767 year, equivalent to 28 days. $r$ is set at 5%; $\sigma$ is 20% per annum. The option prices $c$ are estimated through the Black-Scholes model and are treated as the true option prices. To simulate noise in prices, we generate a set of error terms, $\varepsilon$, which follows the Gaussian distributions $N(0, \sigma^2)$. Let $c_P = c + \varepsilon$ denote the perturbed option prices and they are de-noised to obtain the filtered option prices $c_D$. Option prices can experience sudden extreme price movements and such extreme movements will not necessarily follow a Gaussian distribution. We could guess that maybe as a non-Gaussian distribution. Non-Gaussian noise distributions, one potential method which could be adopted is by Antoniadis (2002) [1]. Sun and Meinl (2012) [7] also proposes a new filtering algorithm that works particularly well when jumps do occur in the underlying price process. The spline method is employed to back out the RND from the option prices. We infer the RND from the original option prices, the perturbed prices, and the de-noised prices, respectively, so that comparisons can be made amongst them. The two series of the estimated RND from the perturbed and de-noised option prices are then used to forecast the out-of-sample prices, which are compared with the true prices from the Black-Scholes model.

### 4. Results

However, despite the large amount of noise, the de-noised volatilities are very close to the true ones. The excellent de-noising ability of the wavelet method is also reflected in the plot of option prices. We
apply the spline method to the true option prices, the perturbed option prices and the de-noised option prices, respectively. In the last part of the experiment, we test the out-of-sample forecast performance using the estimated RND from the perturbed and the de-noised prices. The option price forecasts based on the RND from the de-noised option prices significantly outperform those based on the density from the perturbed prices. Therefore, it is safe to conclude that wavelet de-noising plays an essential role in density estimation and price forecasting in the options market.

5. Conclusion
In this paper, we carry out Monte Carlo simulations to generate noise for option prices and use the wavelet method to filter them out. Therefore, we conclude that the wavelet de-noising technique can substantially improve the density estimation quality and the forecasting abilities of the estimated densities. Our results show that 46.105% of the options yield better forecast performance after the de-noising technique is applied to the options, while the other options do not benefit from the de-noising in terms of the forecasting performance.

References