Entropy Generation Calculation for Turbulent Fully Developed Forced Flow and Heat Transfer of Nanofluids inside Annuli

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Abstract
This paper analytically investigates the entropy generation of Al₂O₃-water nanofluid flow through annuli with uniform heat flux at the inner wall while the outer wall is insulated. The present study has been done for fully developed and turbulent flow condition and single phase approach is used for the nanofluid modeling and the thermophysical properties of the nanofluid are calculated using available correlations. Control volume approach is selected for calculation of the entropy generation. Total entropy generation is calculated for different values of nanoparticles volume fractions at different geometrical ratios and compared with those of the base fluid. The results for the dependency of entropy generation of the nanofluid on geometrical factors such as L/Dₐ and Dₓ/Dᵧ are obtained and compared for different nanoparticles volume fractions. For all of mentioned cases, It has been found that that the irreversibility due to fluid flow (pressure drop) is dominant and adding nanoparticles leads to increased entropy generation.

Keywords: Nanofluid, Entropy generation, Turbulent forced convection, Annulus, Analytical solution

Introduction
Since nanofluids have eminent heat transfer ability, mostly get used in heat exchangers and in recent years have attracted the attention of researchers and different industries. For instance, in transportation industry two of the biggest car factories of the US, GM and Ford, have ongoing nanofluid research projects. Electronics cooling, defense, space, nuclear systems cooling and biomedicine are other nanofluids applications. Electronic devices have resistance and due to passing electronic current, heat gets produced and if the heat produced does not be taken away it definitely would harm the device. Rafati et al. [6] have presented a study on the application of nanofluids in computer cooling systems. They investigated the use of enhanced thermal properties of nanofluids for the cooling of computer microchips. Their nanofluids were combinations of three different volumetric concentrations of silica, alumina and titania suspended in various mixture of deionized water and ethylene glycol. They observed that the largest decrease was for alumina nanofluid, which decreased processor temperature from 49.4°C to 43.9°C for 1.0% of volumetric concentration and flow rate of 1.0 lpm when compared with the pure base fluid with the same flow rate. They also came to conclusion that there should be a balance between volumetric concentration of nanoparticles and the flow rate to satisfy the economy and power consumption of cooling the system. Ijam and Saidur [7] analyzed a minichannel heat sink with SiC-water nanofluid and TiO₂-water nanofluid turbulent flow as coolants. The results showed that enhancement in thermal conductivity by dispersed SiC in water at 4% volume fraction was 12.44% and by dispersed TiO₂ in water was 9.99% for the same volume fraction. There are also examples of nanofluid applications in transportation industry. Leong et al. [8] investigated the performance of an automotive car radiator operated with nanofluids as coolant. It was observed that, about 3.8% of heat transfer enhancement could be achieved with the addition of 2% copper particles in a base fluid at the Reynolds number of 6000 and 5000 for air and coolant, respectively. Low efficiency of engineering thermal systems which use water, ethylene glycol and etc. as their working fluid has made researchers investigating ways to enhance heat transfer ability of the fluid and so increasing the efficiency. One way of enhancing heat transfer of a fluid is increasing its thermal conductivity. However, there are other ways like changing in geometry of the duct in which the fluid flows or changing thermal boundary conditions, increasing thermal conductivity of the fluid is in more interest of researchers. Since solids have higher thermal conductivity than liquids, adding nano-sized particles known as nanoparticles to the liquid increases its thermal conductivity. Many researches have been carried out to investigate the effective thermal conductivity and dynamic viscosity of nanofluids. For instance, Ghanbarpour et al. [1] investigated the thermal properties and rheological behavior of water based Al₂O₃ nanofluid as a heat transfer fluid both experimentally and theoretically. The result showed that for the nanoparticles mass fraction ranging from 3% to 50% and temperature ranging from 293K to 323K thermal conductivity and viscosity of the nanofluid increase from 1.1% to 87% and from 18.1% to 300%, respectively. Also Xuan and Li [2] presented a study on the thermal conductivity of a nanofluid consisting of copper nanoparticles. The measured data showed that adding 2.5-7.5% copper oxide nanoparticles to the water increases its conductivity by about 24-78%.

There are also many researches conducted on nanofluids heat transfer enhancement. For example, Izadi et al. [3] presented a numerical study on developing laminar forced convection of Al₂O₃-water nanofluid in an annulus using single phase approach. Abu-Nada et al. [4]
investigated natural convection heat transfer enhancement in horizontal concentric annuli using water-based nanofluid containing various volume fractions of Cu, Ag, Al₂O₃ and TiO₂ nanoparticles. The results of their study show that addition of the different types and different volume fractions of nanoparticles have different effects on heat transfer characteristics. The also concluded that for high values of Rayleigh number and high L/D ratio, nanoparticles with high thermal conductivity cause significant enhancement of heat transfer characteristics while for intermediate values of Rayleigh number, nanoparticles with low thermal conductivity cause reduction in heat transfer. Akbarinia et al. [5] have studied two phase mixed convection laminar Al₂O₃-water nanofluid flow in an annulus with constant heat flux at both walls. The calculated results show that at a given Reynolds and Grashof number, increasing nanoparticles volume fraction increases the Nusselt number at the inner and outer walls while it does not have any significant effect on the friction factor. Both the Nusselt number and the friction coefficient at the inner wall are more than their corresponding values at the outer wall. In a nanofluid flow, heat transfer development and pressure drop are in an approximate counterbalance. Improvement of the heat transfer properties causes the reduction in entropy generation while the increment in pressure drop gives more irreversibility and exergy loss in systems. For this reason entropy generation in nanofluids has become a concern for researchers. Moghaddami et al. [9] analytically examined the effects of adding nanoparticles on the entropy generation of Al₂O₃-water and Al₂O₃-ethylene glycol nanofluid flows through a circular pipe under uniform wall heat flux thermal boundary condition in both laminar and turbulent regimes. They concluded that adding nanoparticles improves the thermal performance of Al₂O₃-water flow with Reynolds numbers less than 40,000 and Al₂O₃-ethylene glycol flow with Re < 11. The results also showed that adding nanoparticles to a fluid is effective and decreases entropy generation only when heat transfer irreversibility is dominant. Also Mahian et al. [10] have reviewed the entropy generation due to flow and heat transfer of nanofluids in different geometries and flow regimes theoretically. Torabi and aziz [11] presented a study on entropy generation in an asymmetrically cooled hollow cylinder with temperature dependent thermal conductivity and internal heat generation. Mahian et al. [12] have analytically investigated the entropy generation due to mixed convection between two isothermal cylinders where a transverse magnetic field is applied to the system. The result showed that the entropy generation decreases with increases in the magnetic field. In addition, it is found that with decreases in the radius ratio, the effects of MHD (magneto hydrodynamic) flow on the entropy generation are reduced. Mahian et al. [13] have also presented a study on entropy generation due to flow and heat transfer of nanofluids between co-rotating cylinders with constant heat flux on the walls. Recently, Rafie [14] theoretically investigated the entropy generation calculation for laminar fully developed forced flow and heat transfer of Al₂O₃-water and Al₂O₃-ethylene glycol inside annuli when there is a constant heat flux at inner wall and outer wall is insulated. The results of his study showed that when the ratio of the annuli length to its hydraulic diameter (L/Dₜ) exceeds some critical values, adding of the nanoparticles is not efficient. It was also concluded that for each value of the nanoparticles concentration, there is a length ratio (L/Dₜ) at which the entropy generation is minimized.

In the present study the entropy generation of Al₂O₃-water due to turbulent flow and heat transfer, and the effects of determining factors such as geometry and nanoparticles volume fractions are analytically investigated for the thermal boundary condition of uniform heat flux at the inner wall and insulation at the outer wall.

**Problem description**

**Geometrical description of the problem**

Figure 1 shows the considered concentric annulus in which steady, turbulent, thermally and hydrodynamically fully developed nanofluid flow exists. Constant heat flux is applied at inner wall and outer wall is insulated.

![Geometrical configuration of the concentric annulus](image)

**Thermophysical properties of the nanofluids**

For small temperature differences, the thermophysical properties of nanofluids are functions of nanoparticle volume fraction (φ), base fluid and nanoparticles properties. The density of the nanofluids is given by:

\[
\rho_{nf} = (1 - \phi)\rho_{bf} + \phi\rho_p
\]  

(1)

where \(\rho_{nf}\) is the density of nanofluid, \(\rho_{bf}\) is the density of base fluid and \(\rho_p\) is the density of nanoparticles. As mentioned by Buongiomo [15], assuming that the nanoparticles and the base fluid are in thermal equilibrium, the nanofluid specific heat is derived from:

\[
C_{p,nf} = (1 - \phi)C_{p,bf} + \phi C_{p,p}
\]

(2)

\[
\frac{\rho_{nf}}{\rho_{nf}}
\]

Validity of the above equations is verified due to experimental investigations by Pak and Cho [16] and Xuan and Roetzel [17]. Wang et al. [18] has proposed different correlations for thermal conductivity and viscosity of the nanofluids which are as follows:

For Al₂O₃-water nanofluid:

\[
\mu_{nf} = (123\phi^2 + 7.3\phi + 1)\mu_{bf}
\]

(3)
Thermophysical properties of water and $\text{Al}_2\text{O}_3$ nanoparticles are given in Table 1. The above equations are valid when the temperature variation is smaller than 100°C.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Water</th>
<th>$\text{Al}_2\text{O}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density $\rho$ (Kg/m$^3$)</td>
<td>1000</td>
<td>3900</td>
</tr>
<tr>
<td>Heat capacity $C_p$ (J/kg.K)</td>
<td>4180</td>
<td>880</td>
</tr>
<tr>
<td>Thermal conductivity $k$ (W/m.K)</td>
<td>0.6</td>
<td>40</td>
</tr>
<tr>
<td>Viscosity (Pa.s)</td>
<td>0.001</td>
<td>--</td>
</tr>
</tbody>
</table>

**Governing equations**

Heat fluxes are calculated using the following equations:

$$ q_i^* = h_i(T_{si} - T_m) $$  \hspace{1cm} (5)

And

$$ q_o^* = h_o(T_{so} - T_m) $$  \hspace{1cm} (6)

Corresponding Nusselt numbers are as follows:

$$ Nu_i = \frac{h_iD_h}{k_{nf}} $$  \hspace{1cm} (7)

and

$$ Nu_o = \frac{h_oD_h}{k_{nf}} $$  \hspace{1cm} (8)

Where $D_h$ is hydraulic diameter and is calculated by:

$$ D_h = D_o - D_i $$  \hspace{1cm} (9)

Gnielinski [19] has proposed a semi-empirical correlation for calculating Nusselt number for turbulent flow in concentric annular ducts:

$$ Nu = \frac{(f_{ann}/8)Re Pr}{k + 12.7 \sqrt{f_{ann}/8} (Pr^{2/3} - 1)} \left[1 + \left(\frac{D_o}{L}ight)^{2/3}\right]^{2/3} F_{ann}K $$  \hspace{1cm} (10)

Mentioned equation in valid only for $Re > 10^4$.

In the equation above, “Re” stands for “Reynolds number” and is defined by:

$$ Re = \frac{\rho V D_h}{\mu} $$  \hspace{1cm} (11)

The term $f_{ann}$ represents the friction factor and is calculated by:

$$ f_{ann} = (1.8 \log_{10} Re^* - 1.5)^{-2} $$  \hspace{1cm} (12)

where $Re^*$ is given by:

$$ Re^* = Re \frac{(1 + a^2) \ln a + (1 - a^2)}{(1 - a^2) \ln a} $$  \hspace{1cm} (13)

where “a” is the annulus diameter ratio ($D_o/D_i$).

The other terms are as follows:

$$ k_1 = 1.07 + \frac{900}{Re} - \frac{0.63}{(1 + 10Pr)} $$  \hspace{1cm} (14)

$$ K = (Pr/Pr_{tr})^{0.11} $$  \hspace{1cm} (15)

For the boundary condition of “heat transfer at the inner wall and the outer wall insulated” $F_{ann}$ is calculated using the following formula:

$$ F_{ann} = 0.75a^{-0.17} $$  \hspace{1cm} (16)

And for the boundary condition of “heat transfer at the outer wall and the inner wall insulated”, the following formula is used.

$$ F_{ann} = (0.9 - 0.15a^{0.6}) $$  \hspace{1cm} (17)

The first law of thermodynamics for steady state flow in a control volume is expressed as [20]:

$$ \frac{dQ}{dt} = \frac{dW}{dt} = \left(\frac{V_i^2}{2} + gz_2 + u_2 + \frac{p_2}{\rho}\right) (m) $$  \hspace{1cm} (18)

$$ -\left(\frac{V_o^2}{2} + gz_1 + u_1 + \frac{p_1}{\rho}\right) (m) $$

For an incompressible nanofluid, the internal energy (u) is a function of its temperature ($\Delta u = \Delta T$). On the other hand for a control volume which consists of a horizontal annulus of length $L$ with heat fluxes at inner and outer walls the above equation is reduced to:

$$ \int_0^L \left[ q_i + q_o \right] \text{d}x = m \left(\frac{p_2 - p_1}{\rho_{nf}}\right) + \dot{m}c_{nf}(T_2 - T_1) $$  \hspace{1cm} (19)

Where $T_1$ and $T_2$ are the fluid bulk temperature at the inlet and outlet sections of the control volume, respectively. The term $(p_2 - p_1)$ can be calculated from the following equation:

$$ p_1 - p_2 = \frac{fL \rho_{nf} V^2}{D_h} $$  \hspace{1cm} (20)

The second law of the thermodynamics for the mentioned control volume can is given by:

$$ \dot{S}_{gen} = \dot{m} (s_2 - s_1) + \int \frac{\delta Q_i}{T_{si}} + \int \frac{\delta Q_o}{T_{so}} $$  \hspace{1cm} (21)
for an incompressible fluid:
\[ s_2 - s_1 = C_{nf} \ln \left( \frac{T_2}{T_1} \right) \]  
(22)

Therefore:
\[ \dot{S}_{gen} = nC_{nf} \ln \left( \frac{T_2}{T_1} \right) + \int_0^L \frac{Le_{nf} \alpha d_x}{T_{ai}} + \int_0^L \frac{Le_{nf} \alpha d_x}{T_{am}} \]  
(23)

At each section \( T_{ai} \) and \( T_{am} \) are obtained from eqs.7 and 8. For a given inlet conditions, the outlet temperature is calculated from the first law of thermodynamics. The entropy generation is determined by Eq. 24. All of the mentioned equations have been solved simultaneously using Engineering Equation Solver (EES) software. Dimensions of the geometry, nanofluid volumetric flow rate, fluid properties, nanoparticles volume fraction and the values of heat fluxes at inner and outer surface of the annulus are specified as input parameters.

**Validation of the method**

The results for the Nusselt number obtained from Eq. (24) for the fully developed turbulent flow of \( \text{Al}_2\text{O}_3 \)-water nanofluid inside annuli for the thermal condition that there is a constant heat flux at inner wall and outer wall is insulated are compared with the experimental results obtained by Dirker and Meyer [21]. The results are compared in four different situations:

1. \( \text{Al}_2\text{O}_3 \) nanoparticles volume fraction is 1\% and nanofluid Reynolds number is 10000 and Pr number is 6.122 and annulus diameter ratio varies from 1.68 to 5.04 is considered and the results are presented in Figure 2.

2. Annulus diameter ratio is 2.22 and nanofluid Reynolds number is 10000 and \( \text{Al}_2\text{O}_3 \) nanoparticles volume fraction varies from 0%-5\% (See Figure 3).

3. Annulus diameter ratio is 2.22 and \( \text{Al}_2\text{O}_3 \) nanoparticles volume fraction is 0.01 and nanofluid Reynolds number varies from 10000 to 50000 (Figure 4).

4. Annulus diameter ratio is 2.22 and \( \text{Al}_2\text{O}_3 \) nanoparticles volume fraction is 1\% and nanofluid Reynolds number is 10000 and inlet temperature varies from 300K to 340K (See Figure 5).

This comparison allows us to confirm the accuracy of the calculation of the Nusselt Number. As can be seen, there is a good agreement between the results of the used method and those obtained by Dirker and Meyer [21].
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Figure 5. Comparing the result of the present study with the result obtained by Dirker and Meyer [21] for the condition that annulus diameter ratio is 2.22 and Al₂O₃ nanoparticles volume fraction is 1% and nanofluid Reynolds number is 10000 and inlet temperature varies from 300K to 340K.

Slight differences are caused by the approximation taken to calculate the term K in Eq. (12). While the Pr numbers involved in K must be calculated at each annulus section temperature, it has been calculated at the average temperature of the inlet and outlet annulus sections, to avoid complicated calculating procedures.

Results and discussion
In this section, first the effects of adding Al₂O₃ nanoparticles to water on the entropy generation are investigated. In all cases hydraulic diameter of the annulus is 0.008 (Dₜ = 0.008) and the nanofluid flow enters the annulus with the base fluid Reynolds number of 25000 (Reₜf = 25000) and the temperature of 300K and heat transfer rate of 197.5 Watt is applied at the inner wall of the annulus while the outer wall is insulated.

Figure 6 demonstrates the entropy generation rate ratio against the length ratio (L/Dₜ) of the annulus for diameter ratio of 5 (a=5) and different volume fractions of nanoparticles. The entropy generation ratio is defined by:

\[ \text{Entropy Generation Ratio} = \frac{(\dot{S}_{\text{gen}})_{nf}}{(\dot{S}_{\text{gen}})_{bf}} \quad (24) \]

Figure 6 indicates that for all volume fractions and all length ratios entropy generation of the nanofluids are more than that of the base fluid. It must be noted that for laminar flow the situation is reversed (See Rafee [14]). As can be seen in Figure 6, for a constant nanoparticles volume fraction by increasing length ratio entropy generation increases. For a constant length ratio by increasing nanoparticles volume fraction, there is an augmentation in entropy generation. This shows the importance of the flow friction at the turbulent flow regimes.

Figure 6. Variations of entropy generation ratio of Al₂O₃-water nanofluid flow with the length ratio for different nanoparticles volume fraction.

Figure 7 shows the variation of pumping power ratio with nanoparticles volume fraction for the length ratio of 1000. Pumping power ratio is defined by:

\[ \text{Pumping Power Ratio} = \frac{(\text{Pumping Power})_{nf}}{(\text{Pumping Power})_{bf}} \quad (25) \]

As can be seen, by adding 10% of Al₂O₃ nanoparticles to water, pumping power increases for about 64%. Comparing the present result for pumping power with what has been obtained by Rafee [14], it can be concluded that the effect of adding Al₂O₃ nanoparticles to water on pumping power is more significant for laminar flow in comparison with turbulent flow.

Figure 7. Effects of nanoparticles volume fraction on required pumping power.

Figure 8 illustrates the effects of diameter ratio on the entropy generation ratio. As mentioned previously, hydraulic diameter of the annulus under consideration is 0.008 (Dₜ = 0.008) and the base fluid Reynolds number is 25000 (Reₜf = 25000) so the volumetric flow rate is constant for each diameter ratio of study.
These results are presented for 5% volume fraction of the Al₂O₃ nanoparticles. As illustrated, for a constant length ratio, reduction in diameter ratio causes the entropy generation to increase, and that is because of the time when the inner wall diameter approaches to the outer wall diameter ($D_o / D_i \rightarrow 1$), the cross section area of the annulus approaches to zero which means a very narrow passage that has high pressure drop. Therefore the entropy generation due to pressure drop will increase significantly. Also, for a constant diameter ratio, by increasing length ratio entropy generation increases.

Variations of the entropy generation number ($\frac{\dot{T} \dot{s}_{gen}}{\dot{Q}}$) with diameter ratio is shown in Figure 9 for 5% volume fraction of nanoparticles and compared with the same results of the base fluid. The results are presented for the length ratio of 500. As can be seen, for all diameter ratios, the amount of entropy generation number of the nanofluid is higher than that of the base fluid. It means adding Al₂O₃ nanoparticles to water is not effective for turbulent flow of the nanofluid inside annular ducts. Since flow inlet temperature ($T_i$) and heat transfer rate ($\dot{Q}$) are constant for all cases, it can be concluded that for less diameter ratios, entropy generation is more. It means that the entropy generation due to pressure drop is dominant.

Figure 10 illustrates the dependency of the irreversibility to the base fluid flow Reynolds number. It should be noted that for changing the base fluid Reynolds number with the same amount of volumetric flow rate, the velocity, inner and outer diameters should be simultaneously changed. These results are presented for diameter ratio of 0.8 and length ratio of 1000 and nanoparticles volume fraction of 5% and constant volumetric flow rate of 0.1508 ($\frac{\dot{V}}{\dot{Q}}$).

It is evident that for constant geometrical ratios and heat transfer rates, the increase in base fluid Reynolds number will decrease the entropy generation ratio, but this decrement is not significant. As can be seen, 500% increment in the base fluid Reynolds number results in only 11.73% decrement in entropy generation ratio.

Conclusion
The entropy generation of the steady, turbulent, fully developed forced convection flow of Al₂O₃-water nanofluid in an annulus with constant heat flux at the inner wall is analyzed in the present paper. The most important findings are as follows:

For the interested region of Reynolds number (10000<Re<60000), at constant heat transfer rate and constant volume fraction, by increasing the length ratio of the annulus(L/Dₙ), entropy generation ratio increases. Also, for a constant length ratio, by increasing the nanoparticles volume fraction, the entropy generation ratio increases. For all cases entropy generation ratio is more than 1 which means adding Al₂O₃ nanoparticles to water is not effective thermodynamically for the condition that the flow regime is turbulent (10000<Re<60000). As demonstrated for pumping power ratio, only 10%
increment in nanoparticles volume fraction results in 60% increment in pumping power ratio. It can be concluded that the required pumping power is sensitive to the nanoparticles volume fraction. The dependency of entropy generation on diameter ratio is also presented. It is concluded that for a constant length ratio, by decreasing the diameter ratio, entropy generation increases and that is because of the cross section area of the annulus approaches to zero which means a very narrow passage that has high pressure drop. Therefore the entropy generation due to pressure drop will increase significantly. The result of analyzing the dependency of entropy generation on the base fluid Reynolds number for the condition of constant heat transfer rate and constant volumetric flow rate reveals that the entropy generation decreases with increases of the base fluid Reynolds number. According to the results obtained, it can be concluded that, for the condition of turbulent flow inside an annulus, adding Al$_2$O$_3$ nanoparticles to water can enhances the Nusselt number but increase the entropy generation.

**List of Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>diameter ratio, $a = \frac{D_o}{D_i}$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat capacity (J kg$^{-1}$K$^{-1}$)</td>
</tr>
<tr>
<td>D</td>
<td>diameter (m)</td>
</tr>
<tr>
<td>f</td>
<td>friction factor (-)</td>
</tr>
<tr>
<td>h</td>
<td>convective heat transfer coefficient (W m$^{-2}$K$^{-1}$)</td>
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<tr>
<td>k</td>
<td>thermal conductivity (W m$^{-1}$K$^{-1}$)</td>
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<tr>
<td>L</td>
<td>annulus length (m)</td>
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<tr>
<td>$m$</td>
<td>mass flow rate (kg s$^{-1}$)</td>
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<td>Nu</td>
<td>Nusselt number (-)</td>
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<td>P</td>
<td>pressure (Pa)</td>
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<tr>
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<td>Prandtl number (-)</td>
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<td>Re</td>
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<td>temperature (K)</td>
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<tr>
<td>V</td>
<td>velocity (m s$^{-1}$)</td>
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**Greek symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\rho$</td>
<td>density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>nanoparticles volume fraction (%)</td>
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<td>$\mu$</td>
<td>dynamic viscosity (Pa.s)</td>
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**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
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<td>bf</td>
<td>base fluid</td>
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**References**