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Mass Transfer of Blood Oxygen in the Vessel

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Abstract

Mass transfer in the coronary artery plays an important role in the development of atherosclerosis and is one of the most important topics in the pathology of vascular diseases. In this article we studied oxygen transport in the avascular wall of a coronary artery by numerically solving the convection-diffusion equations. The governing equations for mass transfer of oxygen are studied and we could obtain the distribution of oxygen concentration in the vessel and the boundary condition and discussed effect of various factors such as the thickness of the avascular region and blood flow. Oxygen carried by hemoglobin and oxygen consumption in the vessel wall are important physiological parameters in this study. To validate the numerical solution we compare the distribution of the partial pressure of oxygen in the bottom section of the vessel with the current results and suitability is obtained. The results show that with increasing the flow, the minimum oxygen partial pressure in the artery wall is increased and with increasing the thickness of the wall, the minimum oxygen pressure is decreased. These results are useful for predicting the area susceptible to formation of atherosclerosis.

Key words: Oxygen transport, coronary artery, atherosclerosis, hypoxia, numerical methods

Introduction

Atherosclerosis is a disease that with stenosis in the artery caused impairs in blood supply system and thus the tissues does not get enough oxygen. The abnormalities in the supply of oxygen to the wall are linked with the conditions of hyperoxia or hypoxia, which accelerates atherosclerosis by initiating metabolic abnormalities. In Iran, a significant proportion of deaths are due to cardiovascular disease that the main root of this deaths is Atherosclerosis. Thus understanding the significance of a wide range of factors that affect the oxygen transport to the arterial wall becomes critical to investigate the pathogenesis of atherosclerosis

Heughan et al. [1], in an experimental study of oxygen tension in atherosclerotic lesions in rabbits, measured the oxygen tensions in atheromata. They found that the lowest tension in the 2-week old injury was 10 mmHg and it was 12 mmHg for a 5 week old injury. Schneiderman et al. [2] measured the oxygen tensions ex-vivo inside the wall of a rabbit thoracic aorta. They showed that the location of minimum Po_2 in the wall indicates that the mid to inner media would be the most prone to hypoxic injury if oxygen transport from the lumen were to become impaired. Jurrus and Weiss [3] measured in-vivo, the oxygen tension in normal tissues and atherosclerotic lesions. Unlike Heughan et al. [1], who found oxygen tension profile to be discontinuous at intima, Jurrus and Weiss [3] found that the oxygen tension profiles were smooth continuous curves. They found that the minimum Po_2 decreases from 55 mmHg for the thinnest tissue to zero for 720 micron tissue thickness. Crawford et al. [4] experimentally measured the Po_2 in vivo in dog femoral arteries. They concluded that a significant Po_2 gradient exists between free stream arterial blood and the intima, forming a measurable lumen boundary layer. They further confirmed their earlier finding that the lumen side resistance may be a significant determinant of arterial wall oxygenation. Zemplenyi et al. [5] experimentally showed that the in-growth of *vasa vasorum* counteracts the impairment of oxygen supply caused due to subintimal thickening. Thus, it is

an important mechanism against hypoxia and may be an essential protective factor in atherogenesis.

Back [6] numerically investigated the transport of oxygen to the wall from the blood flowing in a converging-diverging tube without an oxygen consuming wall at a velocity averaged over a cardiac cycle. This study found that there is an increase in oxygen transport to the wall in the accelerated flow regions, while it decreases in decelerated flow regions. Also, the oxygen transport was found to occur primarily across the cell free plasma layer adjacent to the endothelium. Back [7] numerically analyzed the coupled relationship between oxygen transport in the lumen and in the inner wall. The important conclusion of this study was that the lumen side resistance may be more than the avascular wall side resistance, i.e. the hemodynamics primarily controls the oxygen supply to the arterial wall. Back et al. [8] numerically studied the transport of oxygen to the arterial wall in multiple non-obstructive plaque regions over the complete cardiac cycle. They found that the oxygen transport to the wall varied considerably over the cardiac cycle. This study further confirmed that the lumen side resistance is at least an order of magnitude greater than the wall side resistance at the incipient separation locations; while at the reattachment, the lumen side resistance is equal or less than the wall side resistance.

Schneiderman and Goldstick [9] numerically calculated the thickness of the oxygen concentration boundary layer in the lumen in the presence of the oxygen consuming wall. They noted that significant oxygen diffusion gradients extend into the flowing blood well beyond any luminal plasma layer as also found by Back et al. [8], contradicting the earlier finding of Back [6]. Schneiderman et al. [10] numerically studied the mass transport to the walls of stenosed arteries without considering the oxygen consuming wall thickness. They assumed a constant oxygen concentration along the wall. They found that there are regions of both enhanced and impaired mass flux in the diverging part of a constriction. The critical limitation of this study was that it did not consider the effect of oxygen consuming wall thickness and the effect of nonlinear oxygen binding capacity of hemoglobin. Schneiderman et al. [11] studied the effect of pulsatility on the oxygen transport to the oxygen consuming wall by superimposing Womersley type pulsatility on fully-developed flow. They concluded that the pulsatile flow negligibly affects the oxygen transport to the wall.

Rappitsch and Perktold [12] numerically studied the oxygen transport in a stenosed artery at a constant flow rate with shear rate dependent and constant permeability of the wall. They found that in the flow region downstream of the stenosis, oxygen concentration decreases to 75% of the inlet concentration and the wall flux also reduces in this region, thus confirming findings of Back et al. [8]. In another study by the same authors [16], analyzing the transport of albumin in a stenosed artery, they concluded that the endothelial resistance is more compared to the lumen side boundary layer resistance.

Moore and Ethier [13] numerically studied the transport of oxygen in large arteries taking into consideration important physiological factors such as the oxygen consuming wall thickness and nonlinear oxy-hemoglibion saturation curve. They found the Po_2 curve to be continuous showing a luminal oxygen concentration boundary layer and a minimum Po_2 in the medial region. The important conclusion of this study was that the oxygen transport is primarily determined by the wall-side effects and the hemodynamics plays a secondary role in oxygen transport to the wall. Stangeby and Ethier [14], in a recent study, numerically analyzed the transport of macromolecules including porous oxygen consuming wall at a steady flow of blood in the lumen. They found that the mass flux to the wall increases in the acceleration region and it decreases in the deceleration region reaching minimum at the point of flow separation.

Thus, the conclusions from the previous studies, which are well accepted are: 1) The oxygen concentration boundary layer in the lumen extends well beyond the single plasma layer; 2) The radial oxygen concentration curve is continuous from the middle of the lumen till the *vasa vasorum*; and 3) The oxygen flux to the wall increases in the acceleration region and decreases in the deceleration region.

In this study steady and laminar flow of the blood in an unstenosis straight vessel, with assuming a Newtonian and non- Newtonian fluid, is solved numerically. Also the governing equations for mass transfer of oxygen inside the vessel and its walls have been solved.

Methodology

Geometry

"Figure 1" shows the geometry used for the analysis in an unstenosed coronary artery. It is an axi-symmetric representation of an artery of diameter, d_e = 3 mm. The thickness of avascular region, δ, is 300 micron and axial length is 7 cm.

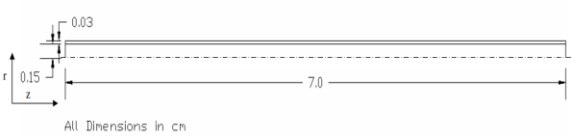

Figure 1. Geometry showing straight artery

Problem formulation

The oxygen concentration distribution in an artery is determined by the velocity field. Hence, in order to establish the velocity distribution, the following continuity and momentum conservation equations are solved.

Continuity equation:

$$\rho\nabla.(\vec{v}) = 0 \quad (1)$$

Where ρ is the density of blood and $\vec{v}$ is the velocity vector.

Momentum conservation equation:

$$\rho\frac{\partial}{\partial t}(\vec{v}) + \rho\nabla.(\vec{v}\vec{v}) = -\nabla p + \nabla.(\bar{\bar{\tau}}) \quad (2)$$

Where p is the static pressure and $\bar{\bar{\tau}}$ is the stress tensor. The stress tensor $\bar{\bar{\tau}}$ is given by,

$$\bar{\bar{\tau}} = \mu[(\nabla.\vec{v}) + \nabla.\vec{v}^T)] \quad (3)$$

where μ is the viscosity.

Taking into account the oxygen carried by the hemoglobin and that dissolved in plasma, the oxygen mass conservation equation in the lumen becomes

$$\frac{\partial}{\partial t}(c+\gamma) + u\frac{\partial}{\partial z}(c+\gamma) + v\frac{\partial}{\partial r}(c+\gamma) = D_b\left[\frac{\partial^2 c}{\partial r^2} + \frac{1}{r}\frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial z^2}\right] \quad (4)$$

where, c, is the oxygen concentration in ml_o/ml_{blood} and γ is the oxygen carried by the hemoglobin. The terms on the left hand side of the equation represent variation with time and the convective transport. The terms on the right hand side of the equation represent the diffusive transport. Since the hemoglobin is carried only by convection and not by diffusion, the term γ is absent from the right side of the equation. The oxygen concentration, c, in this equation, is related to Po_2 by the solubility relation.

$$c = \alpha.Po_2 \quad (5)$$

where, α, is the solubility coefficient for oxygen.

The lumen side oxygen transport equation is coupled with oxygen transport and consumption in the avascular wall. The oxygen concentration conservation equation in the wall region is

$$\frac{\partial c}{\partial t} = \nabla.(D_w\nabla c) - \dot{q} \quad (6)$$

where, $\dot{q}$, is a constant volumetric consumption rate of oxygen by the cells within the wall region.

Material properties

The density of the blood is 1.05 g/cm^3. The diffusivity of oxygen in the blood, D_b , and that in the wall region, D_w , is taken to be 1×10^{-5} cm^2/s. The value of the solubility coefficient for oxygen in blood and in the wall region is 3×10^{-5} $ml_o/ml-mmHg$ [14]. The value of the constant volumetric consumption rate of oxygen in the arterial wall, $\dot{q}$, is 1.3×10^{-4} $ml_o/ml_{tissue}-s$ [4].

Boundary conditions

Velocity boundary condition: For velocity distribution calculations, no-slip boundary condition is applied at the lumen-wall interface. No radial flow boundary condition is applied at the axis. Constant flow rates of 50 (basal), 100 and 180 ml/min (hyperemic), with fully developed parabolic profile for axial velocity, are applied at inlet.

Species boundary condition: For species distribution, uniform concentration of oxygen, corresponding to normal blood Po_2 of 95 mmHg, is applied at inlet [12]. At *vasa vasorum* oxygen concentration corresponding to $Po_{2,V}$ of 45 mmHg is applied [4]. The oxygen concentration is calculated using Eq. (5). Zero flux boundary condition is applied at the axis along the lumen. The oxygen transport from the blood in the lumen to the wall has continuity of species and flux across the endothelial wall.

Results and discussion

To check the accuracy of the numerical computations, a simulation was done to replicate the results obtained by Schneirderman et al. [11]. The geometry, material properties and boundary conditions used for this simulation were the same as used by Schneirderman et al. [11]. "Figure 2" shows the radial oxygen partial pressure profiles obtained by these two studies.

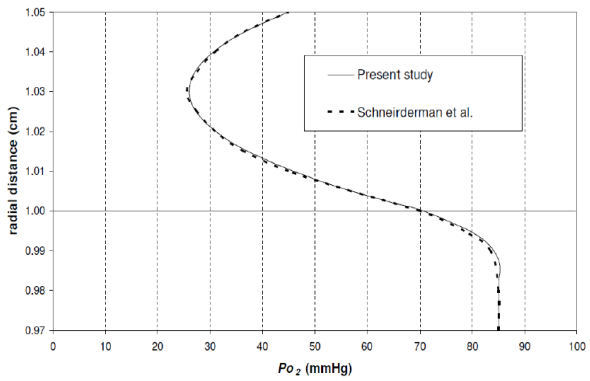

Figure 2. Comparison between results obtained by the present study and those obtained by Schneirderman et al. [11] using same geometry, properties and boundary conditions

"Figure 3" shows radial variation of Po_2 at axial position, z, of 7 cm from inlet and δ = 300 µm for basal to hyperemic flows. The Po_2 decreases near the endothelial wall and the gradient causes transport of oxygen from the lumen blood to inside avascular region of the wall. The Po_2 reaches minimum value, $Po_{2,min}$, in the wall where the radial flux becomes zero. For basal flow $Po_{2,min}$ reaches 4.7 mmHg. Subsequent to the minima, the Po_2 increases radially outward towards *vasa vasorum* as it acts as an additional source of oxygen. Oxygen concentration boundary layer of ~80 µm is observed in the lumen near the wall.

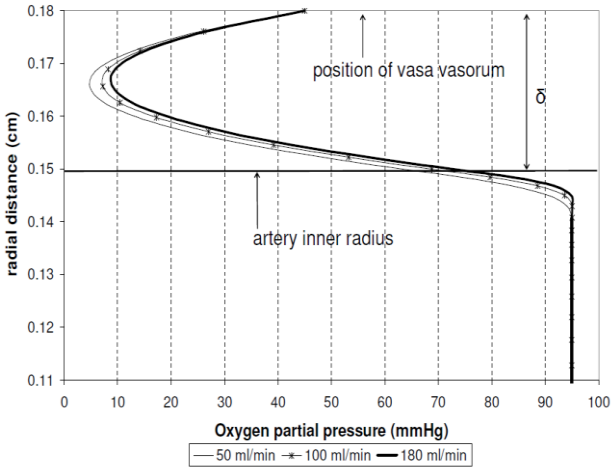

Figure 3. Po_2 along the radial distance for Q = 50, 100, 180 ml/min, δ = 300 µm at axial distance z = 7 cm from inlet

"Figure 4" shows the variation of the $Po_{2,w}$ with axial length for avascular thickness of δ = 300 µm from basal to hyperemic flows. The $Po_{2,w}$ is lower for basal flow because of the lower oxygen flux to the endothelial wall. The relatively lower velocity gradients for basal flow result in lower flux to the wall.

"Table 1" and "Figure 5" show the effect of avascular thickness on various parameters such as $Po_{2,w}$, $Po_{2,min}$ and δ_b. The avascular thickness has significant effect on the Po_2 profile in the wall. For δ = 300 µm and basal flow rate the $Po_{2,min}$ reaches lowest value of 4.7 mmHg as compared to 33.6 mmHg for δ =200 µm because of the higher consumption in the wall. The change of wall thickness does not have any effect on oxygen concentration boundary layer, δ_b. The change of flow rate has significant effects on $Po_{2,min}$. For 300 micron wall thickness $Po_{2,min}$ increases from 4.7 mmHg for basal flow to 9.0 mmHg for hyperemic flow.

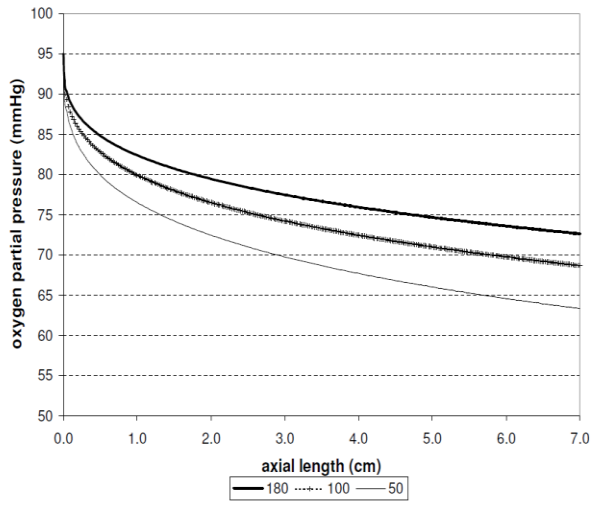

Figure 4. Variation of $Po_{2,w}$ with axial length. δ = 300 µm

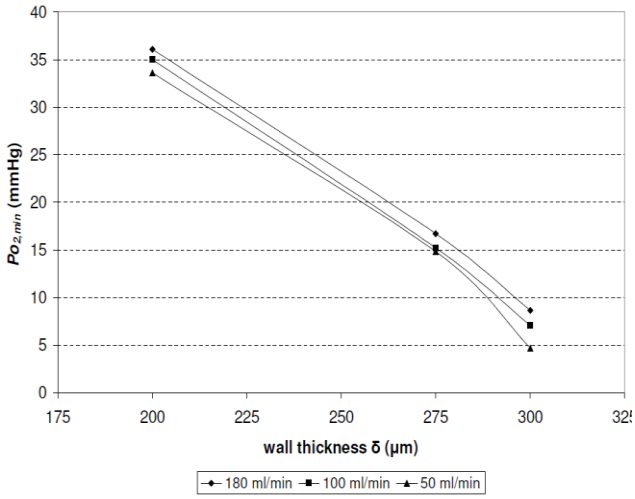

Figure 5. Variation of $Po_{2,min}$ for variable δ at z = 7 cm

Table1. Effect of wall thickness on various parameters in an unstenosed artery

Q ml/min	$Po_{2,min}$ mmHg	$Po_{2,w}$ mmHg	δ_b µm
δ = 200 µm, z =7 cm			
50	33.6	69.5	80
100	35	73.7	60
180	36	77	50
δ = 300 µm, z = 7cm			
50	4.7	63.3	80
100	7.1	68.6	60
180	9	72.6	50

"Figure 6" shows the radial Po_2 profiles at the location of flow reattachment with constant blood viscosity and Carreau model viscosity. The $Po_{2,min}$ with constant viscosity is 10.7 mmHg while that with Carreau model is 9.0 mmHg (difference of 18.8%). The Po_2 at the endothelium $Po_{2,w}$ with constant blood viscosity is 77.61 mmHg while that with Carreau model viscosity is 73.37 mmHg (difference of 5.7%). Thus it can be concluded that the effect of non-Newtonian blood viscosity should be included while calculating the oxygen transport to the arterial wall.

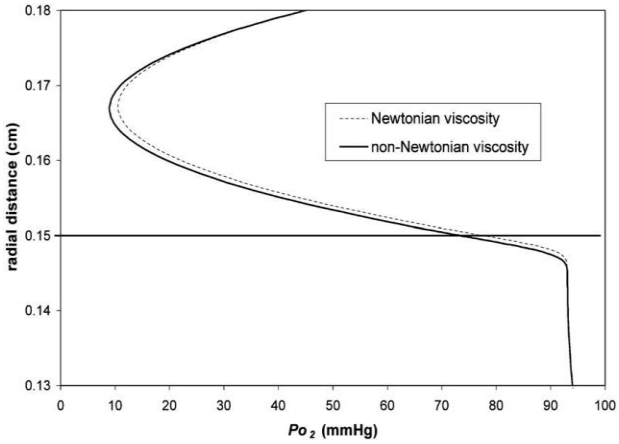

Figure 6. Radial P_{O_2} profile at the location of flow reattachment with Newtonian and non-Newtonian blood viscosity. Flow rate = 180 ml/min. Oxygen consumption in the wall = 1.3×10^{-4} ml_o/ml_{tissue}-sec. The wall thickness is 300 micron.

Conclusion

This study presents an analysis of the oxygen transport to the avascular wall of an unstenosed coronary artery. Important physiological aspects, such as oxygen carried by the hemoglobin, and consumption of oxygen in the wall, are accounted for. The radial Po_2 profiles show that the Po_2 curve is continuous from the arterial lumen to the *vasa vasorum*. The lumen side oxygen concentration boundary layer is ~80 micron thick. The location of

the medial region of the arterial wall for a normal as well as stenosed artery always has a low value of Po_2. Thus, this region is highly susceptible to hypoxic injury. According to the figures we found that the region for hypoxic is middle of the lumen. Also increasing of the thickness and decreasing of the flow will increase the hypoxic injury.

List of Symbols

c –oxygen concentration (ml_o/ml_{blood});
d_e –normal artery mean diameter (mm);
D_b–diffusivity of oxygen in blood (cm^2/s);
D_w –diffusivity of oxygen in the wall region (cm^2/s);
p –static pressure ($dyne/cm^2$);
Po_2 –partial pressure of oxygen (mmHg);
$Po_{2,\min}$ –minimum partial pressure of oxygen in the avascular wall (mmHg);
$Po_{2,w}$–partial pressure of oxygen at the lumen–endothelium interface (mmHg);
$Po_{2,v}$–partial pressure of oxygen at the outer surface of the arterial wall (mmHg);
$\dot{q}$ –volumetric consumption rate of oxygen by the cells within the wall region (ml_o/ml_{tissue}-sec);
r –radial distance (cm);
u –axial component of velocity (cm/s);
v –radial component of velocity (cm/s);
$\bar{v}$ –velocity vector (cm/s);
z –axial distance (cm);

Greek symbols

δ –thickness of the avascular wall (micron);
δ_b –Lumen side oxygen concentration boundary layer thickness (micron);
ρ –density of the blood (g/cm^3);
$\bar{\bar{\tau}}$ –stress tensor ($dyne/cm^2$);
γ –oxygen carried by hemoglobin (ml_o/ml_{blood});
α –solubility coefficient for oxygen (ml_o/ml_{blood} - mmHg).

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